Dynamics of Disease Models with Self-Diffusion: A Study of Cholera in Ghana

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Introduction

- Reaction-diffusion (RD) systems involve constituents being locally transformed into each other by chemical reactions and transported in space by diffusion.
- **Cholera** is an infectious disease of the small intestine caused by the gram-negative bacterium, ***Vibrio cholerae***.
- Left untreated, one suffers severely from diarrhoea and vomiting, causing rapid dehydration and electrolyte imbalance and even death.

Important Problem

- In Ghana, the outbreak of Cholera still posses a public health threat
- While time-dependent models exist, spatiotemporal models are better at explaining the prevalence of the disease
- A 2D reaction-diffusion model incorporating both human and environmental factors explains the dynamics and emerging patterns of the disease spread

Compartmental Model for Cholera

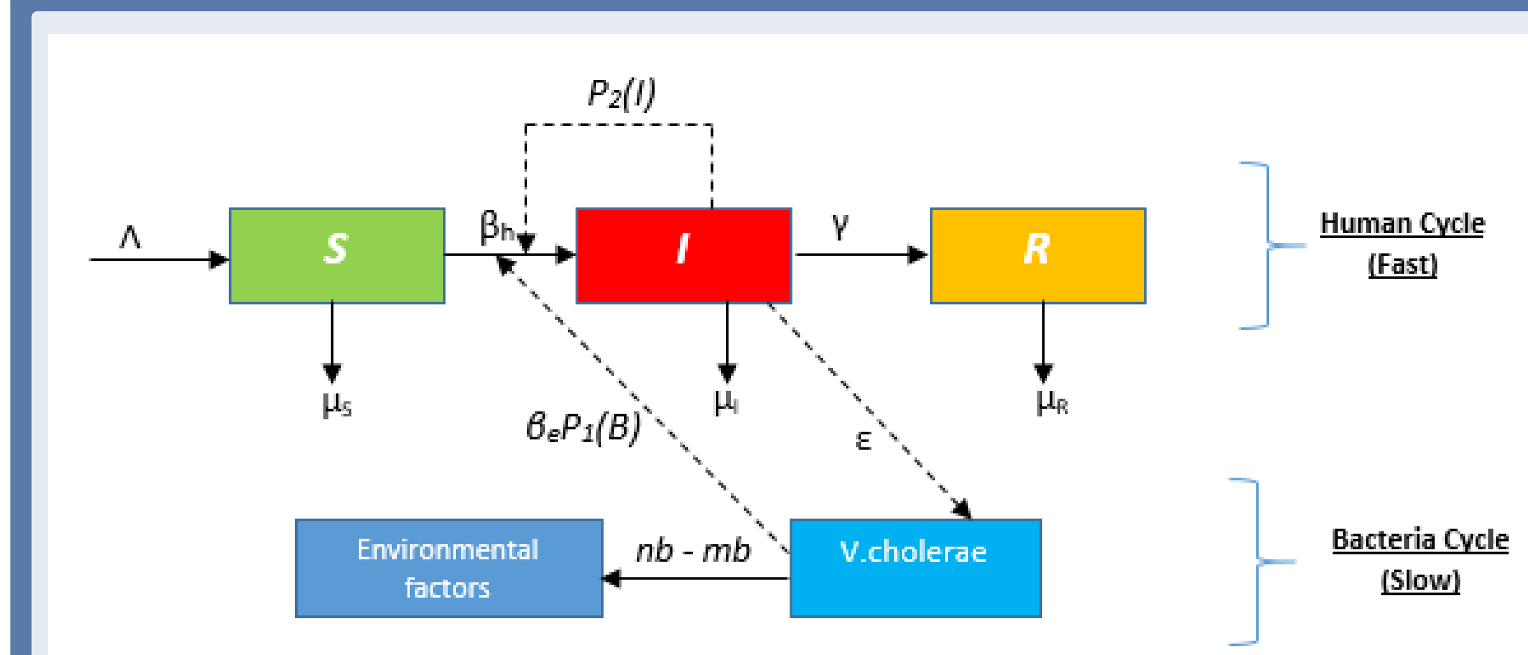

Figure : SIR-B Compartmental Model

$$\frac{dS}{dt} = \Lambda N - \mu S - \beta_e \frac{B}{B+k} S - \beta_h SI \qquad (1)$$

$$\frac{dI}{dt} = \beta_e \frac{B}{B+k} S + \beta_h SI - (\mu + \alpha + \gamma) I \qquad (2)$$

$$\frac{dR}{dt} = \gamma I - \mu R \qquad (3)$$

$$\frac{dB}{dt} = \epsilon I - (nb - mb) B \qquad (4)$$

where:

$$P_1(B) + P_2(I) = \beta_e \left(\frac{B}{B+k}\right) + \beta_h I : \text{force of infection} \qquad (5)$$

Par.	Meaning	Par.	Meaning
β_e	Contact Rate (env't-to-human)	nb	Growth rate of bacteria
β_h	Contact Rate (human-to-human)	mb	Loss rate of bacteria
k	Conc. of bacteria	Λ	Human birth rate
γ	Recovery rate of I	μ	death rates
ϵ	I adding bacteria to source	α	induced death rate of I

Table : Parameters used in equations (1)-(4)

Reaction-Diffusion System

$$\frac{\partial S}{\partial t} = \Lambda N - \mu S - \beta_e \frac{B}{B+k} S - \beta_h SI + D_1 \nabla^2 S \qquad (6)$$

$$\frac{\partial I}{\partial t} = \beta_e \frac{B}{B+k} S + \beta_h SI - (\mu + \alpha + \gamma) I + D_2 \nabla^2 I \qquad (7)$$

$$\frac{\partial B}{\partial t} = \epsilon I - (nb - mb) B + D_3 \nabla^2 B \qquad (8)$$

s.t. zero-flux Neumann boundary conditions and $D_1 \neq D_2 \neq D_3$ else a linearly stable uniform steady state of **S,I,B**

Initial conditions: $S(0) = S_0 > 0,\ I(0) = I_0 > 0,\ R(0) = R_0 = 0$

Result 1

Basic Reproduction Number (R_0):

- $R_0 < 1$, disease dies out: Stable DFE.
- $R_0 > 1$, epidemic: Unstable DFE.
- $R_0 = 1$, endemic

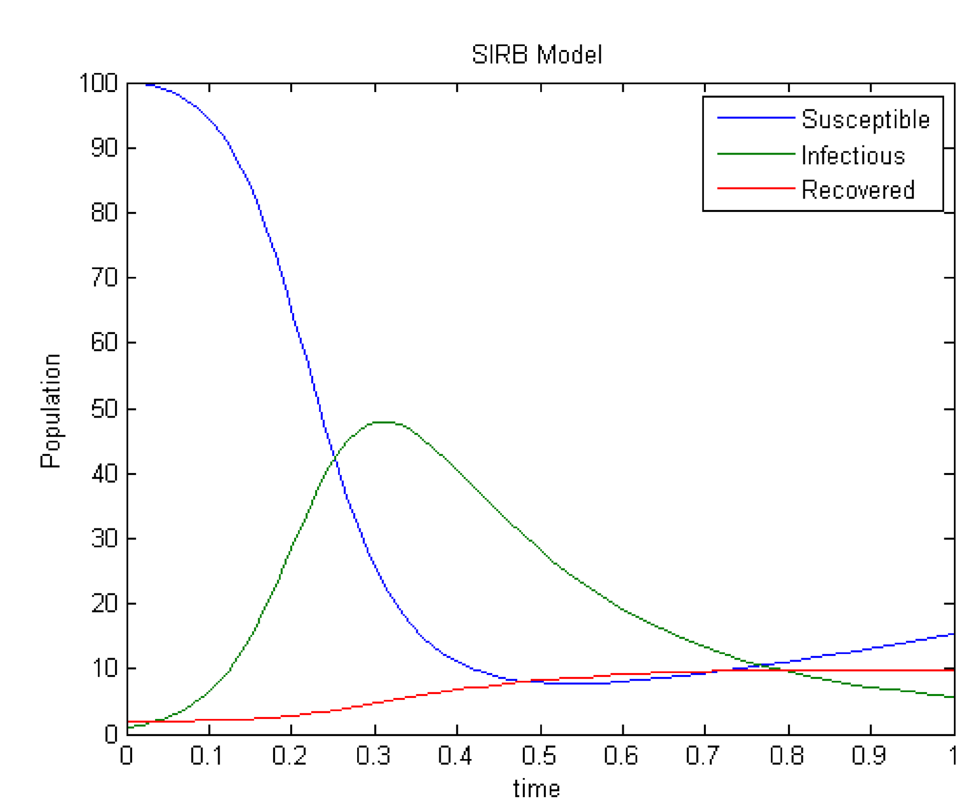

Figure : SIRB 1-D for which the model predicts there is an epidemic.

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Result 2

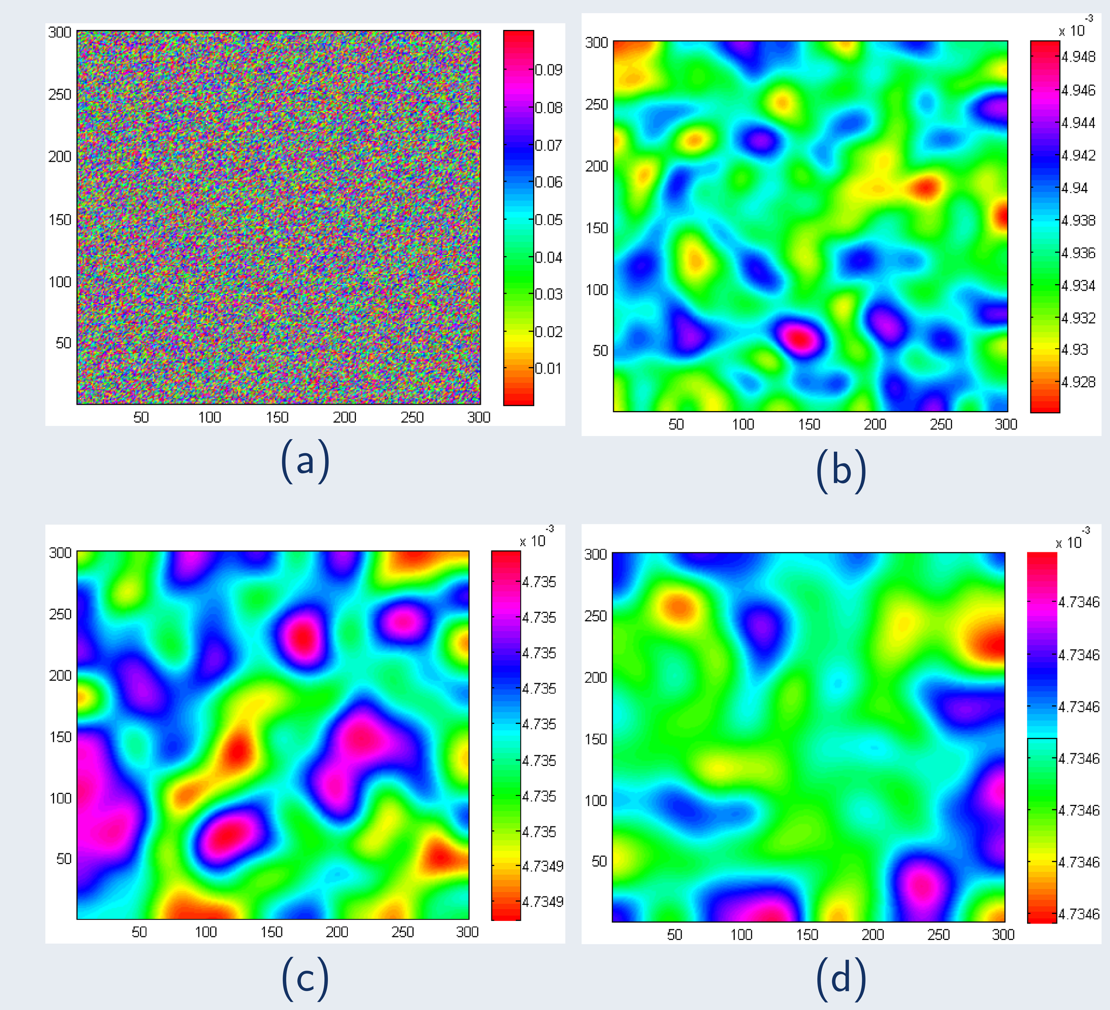

Figure : Spatial distribution of infectious population density of the model system (6) - (8). $D_2 = 0.25$. Spatial pattern are obtained at different time levels: plot for (a)T=0 (b)T=50 (c)T=80 (d)T=100

Conclusion

- Disease prevalence reduces with a decreasing R_0 and vice versa.
- The RD system saw the inclusion of diffusive terms to explicitly track the movement of human host and the bacterial concentration in a heterogeneous environment.
- Diffusion accounts for the spread of an epidemic as individuals tend to diffuse in the direction of lower densities of the disease

References

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