Center of Mass Motion and the Mott Transition in Light Nuclei

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Introduction

- One of the most important topics in nuclear physics is to study the composition, stability, and the dynamics of finite nuclear matter (nuclei) under different conditions.

- At very low densities nuclear matter is often viewed as a vapour of widely separated nucleons.

Introduction

- Studies indicate that light nuclei (clusters) are formed in the vapor at very low vapour densities to minimize the energy [1-3].

- At a certain density of nucleons in the surrounding vapour (**Mott density**) the cluster dissolve and become part of the surrounding vapour due to **Pauli blocking effect** [4].

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[1] Röpke G., et al., 1982 Nucl. Phys. A 379
[2] Röpke G., et al., 1983 Nucl. Phys. A 399
[3] Jaqaman H R 1988 Phys. Rev. C 38
[4] Röpke G, 2009 Phys. Rev. C 79

Research Problem

- In our work we will study the effect of including the center-of-mass (**CM**) momentum of light cluster on the **Mott density**.
- All previous calculations ignored CM motion (They put $K = 0$)
- we will find the **Mott density** of light clusters up to $A = 4$ (^2H: deuteron, ^3He: helion, ^3H: triton, ^4He: alpha) at different temperatures .

^{3}He-Neutron System

- We will consider ^{3}He clusters (**helions**) moving in a hot low-density vapour of protons and neutrons.

- For simplicity, we derive the wave function of a system composed only of one ^{3}He cluster and a free neutron confined in a cubic box of edge **L**.

$$\boxed{\Psi(1234) = \psi_{space}(1234)\psi_{spin}(1234)}$$

Wave Function of ^{3}He

- Using Harmonic Oscillator (HO) nuclear shell model the space wavefunction of an isolated **helion** moving with momentum $\hbar\vec{K}$ inside a cubic box of edge L is

$$\boxed{\psi_{space}(123) = Ae^{-\frac{\beta^2}{2}\left[(\vec{r}_1-\vec{R})^2+(\vec{r}_2-\vec{R})^2+(\vec{r}_3-\vec{R})^2\right]}\frac{1}{L^{3/2}}e^{i\vec{K}.\vec{R}}}$$

A is the normalization constant, $\vec{R} = \dfrac{\vec{r}_1+\vec{r}_2+\vec{r}_3}{3}$

$\beta^2 = m\omega/\hbar$ where ω is the angular frequency of the HO.

$\vec{r}_1, \vec{r}_2, \vec{r}_3$ are the position vectors of the nucleons within ^{3}He (the two protons and the neutron respectively).

Wave Function of ^{3}He-Neutron System

- So

$$\boxed{\psi_{space}(1234) = \frac{1}{27^{1/4} L^{3/2}} \left(\frac{\beta}{\sqrt{\pi}}\right)^3 e^{-\frac{\beta^2 r^2}{4}} e^{\frac{2}{3} i \vec{K}.\vec{\rho}} e^{-\frac{\beta^2}{3}(\vec{r}_3 - \vec{\rho})^2} e^{i\frac{\vec{K}}{3}.\vec{r}_3} \frac{1}{L^{3/2}} e^{i\vec{k}.\vec{r}_4}}$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{\rho} = \dfrac{\vec{r}_1 + \vec{r}_2}{2}$

$\vec{r}_4$ is the position vector of the free neutron.

$\vec{k}$ represents the wave vector of the free neutron.

$\vec{K}$ represents the wave vector of the ^{3}He nucleus.

Wave Function of ^{3}He-Neutron System

The total wavefunction is antisymmetiric in the two neutrons

$$\Psi_{tot}(1234) = \sqrt{\frac{1}{4}}\left\{\psi_{space}^{sym}(1234)\sum_{m=-1}^{1}\chi_{1\,m}(3,4) + \psi_{space}^{anti}(1234)\sum_{m=-1}^{1}\chi_{0\,0}(3,4)\right\}$$

3 triplet spin states for the two neutrons

1 singlet spin state for the two neutrons

Binding Energy of ^{3}He-Neutron System

$$B = B_0 - \langle \Psi_{total} \mid v_{14} + v_{24} + v_{34} | \Psi_{total} \rangle - \langle \Psi_{total} | v_{124} + v_{134} + v_{234} | \Psi_{total} \rangle$$

B_0 binding energy of an isolated ^{3}He nucleus.

v_{14}, v_{24}, v_{34} and $v_{124}, v_{134}, v_{234}$ represent the two- and three-body interactions between the bound nucleons and the free neutron.

Using Skyrme force [1] →

$$B = B_0 - \frac{t_0}{L^3}\left[\left(\frac{1}{2} + x_0\right) + \left(1 + \frac{x_0}{2}\right)\frac{1}{\sqrt{27}}\left(\frac{6}{\sqrt{7}}\right)^3 e^{-\frac{6}{7\beta^2}\left(\frac{\vec{K}}{3} - \vec{k}\right)^2}\right] + \frac{t_3}{L^3}\frac{\beta^3}{\pi^{3/2}}\left[\frac{1}{\sqrt{8}} + \frac{1}{2} e^{-\frac{3}{4\beta^2}\left(\frac{\vec{K}}{3} - \vec{k}\right)^2}\right]$$

[1] Skyrme, T. 1959 *Nuclear Physic* 9(4).

Binding Energy of ^{3}He Immersed in A Vapour of Nucleons

- Considering ^{3}He nucleus immersed in a very low density (ρ) vapour of nucleons.

- Multiplying by the number of nucleons $n = \rho L^3$

$$B = B_0 + \rho \left\{ -t_0 \left[\left(\frac{1}{2} + x_0 \right) + \left(1 + \frac{x_0}{2} \right) \frac{1}{\sqrt{27}} \left(\frac{6}{\sqrt{7}} \right)^3 e^{-\frac{6}{7\beta^2}\left(\frac{\vec{K}}{3} - \vec{k}\right)^2} \right] + t_3 \frac{\beta^3}{\pi^{3/2}} \left[\frac{1}{\sqrt{8}} + \frac{1}{2} e^{-\frac{3}{4\beta^2}\left(\frac{\vec{K}}{3} - \vec{k}\right)^2} \right] \right\}$$

Expectation Value of the Binding Energy

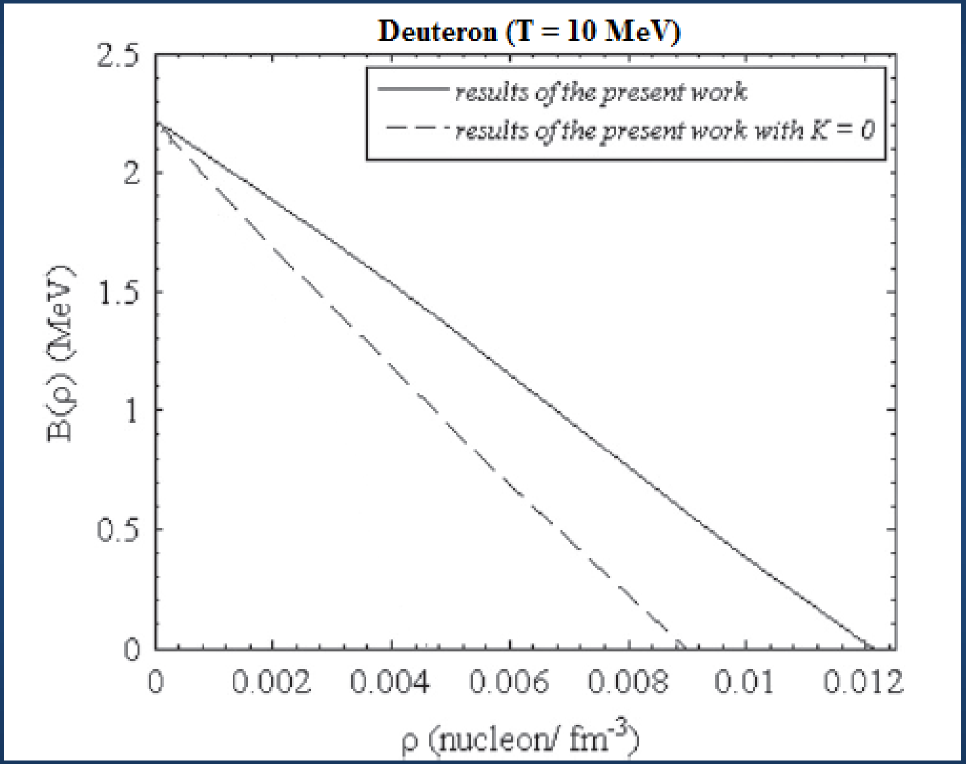
- Assuming thermal equilibrium between ^{3}He nuclei and the surrounding nucleons. Thus, we will take the thermal ensemble average over all values of $\vec{K}$ and $\vec{k}$ using the Fermi-Dirac statistics.

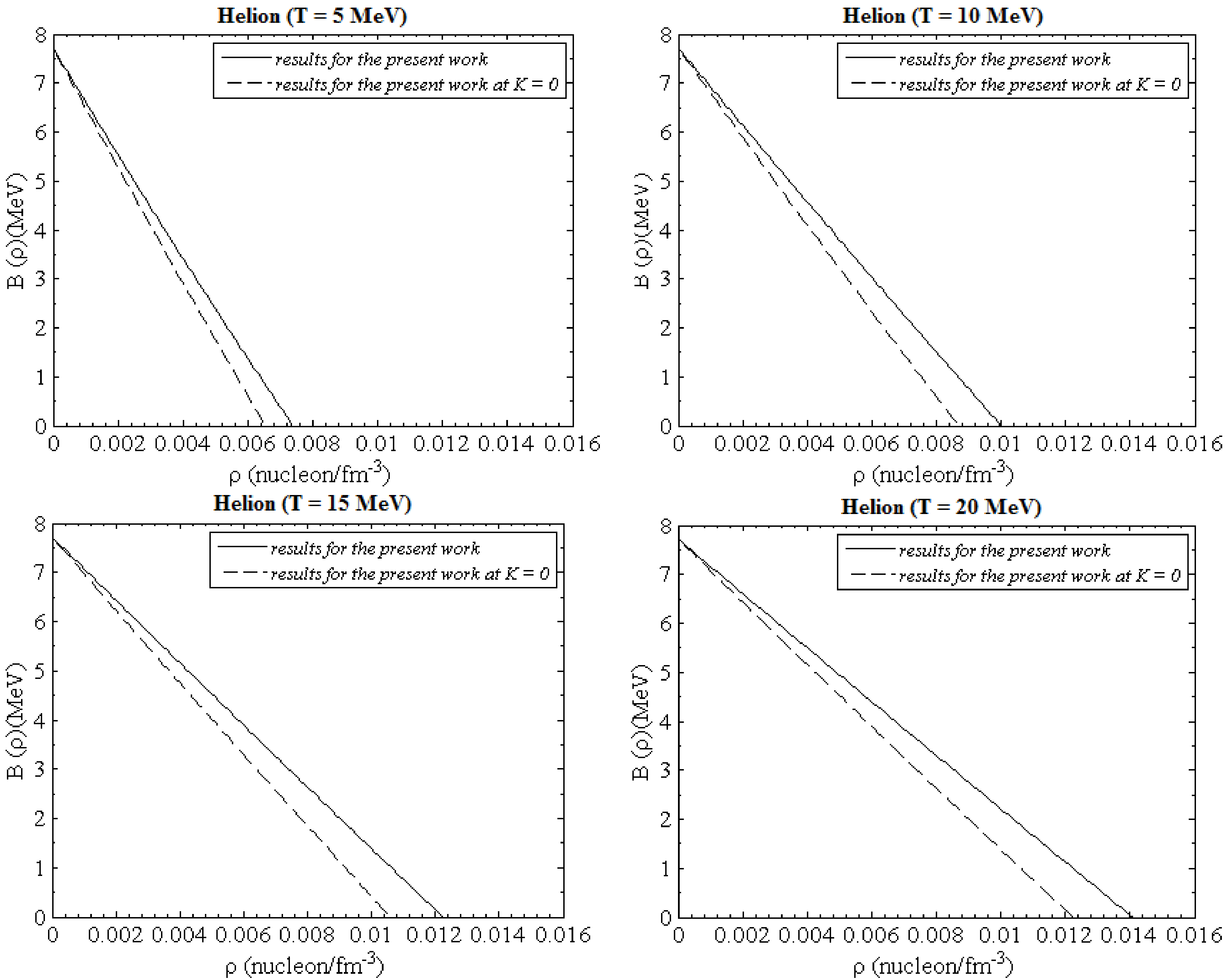
$$\langle B(\rho)\rangle = B_0 + \rho\left\{-t_0\left[-\left(\frac{1}{2}+x_0\right)+\left(1+\frac{x_0}{2}\right)\frac{1}{\sqrt{27}}\left(\frac{6}{\sqrt{7}}\right)^3\langle e^{-\frac{6}{7\beta^2}\left(\frac{\vec{K}}{3}-\vec{k}\right)^2}\rangle\right] + t_3\frac{\beta^3}{\pi^{3/2}}\left[\frac{1}{\sqrt{8}}+\frac{1}{2}\langle e^{-\frac{3}{4\beta^2}\left(\frac{\vec{K}}{3}-\vec{k}\right)^2}\rangle\right]\right\}$$

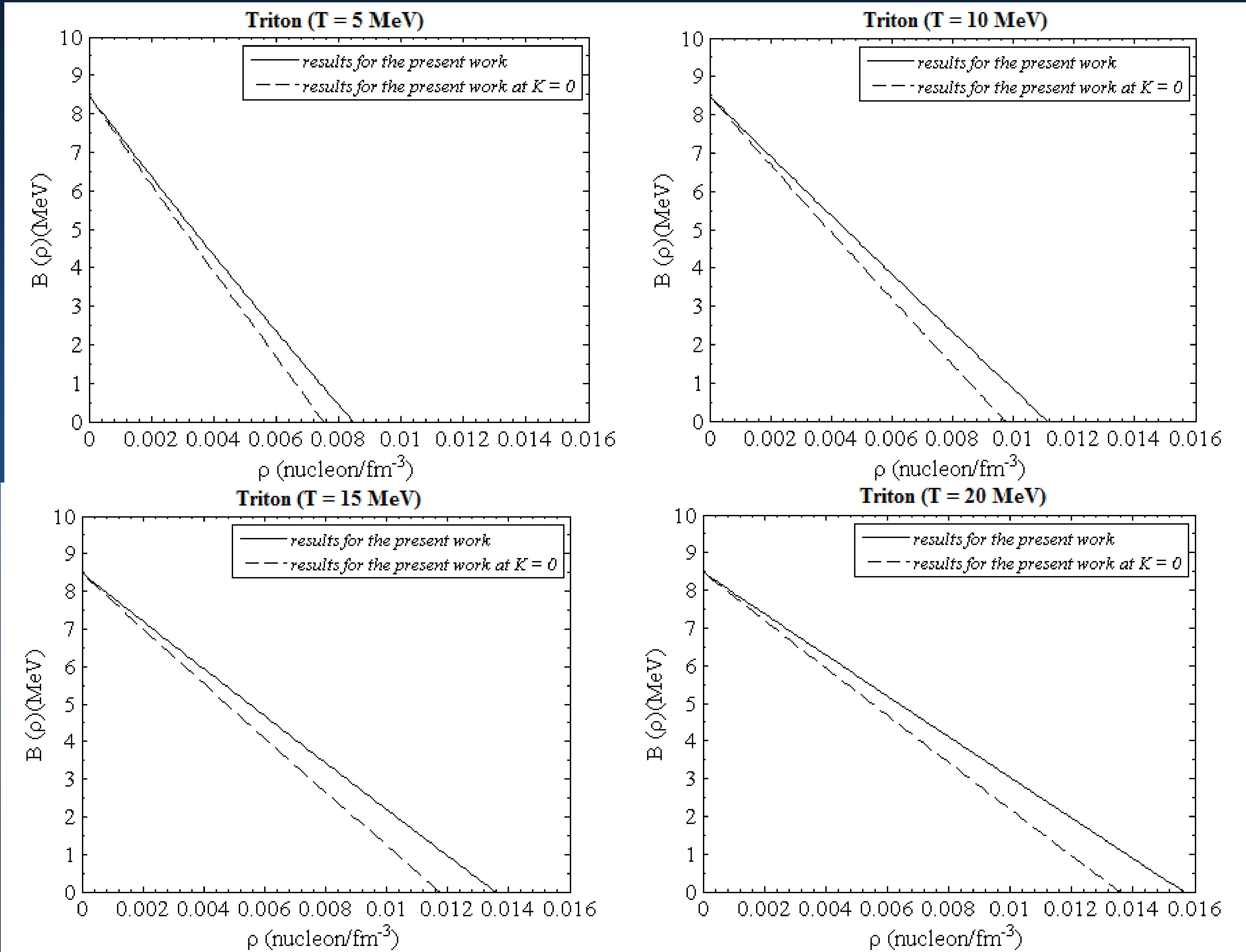
The same procedure is used for other light clusters

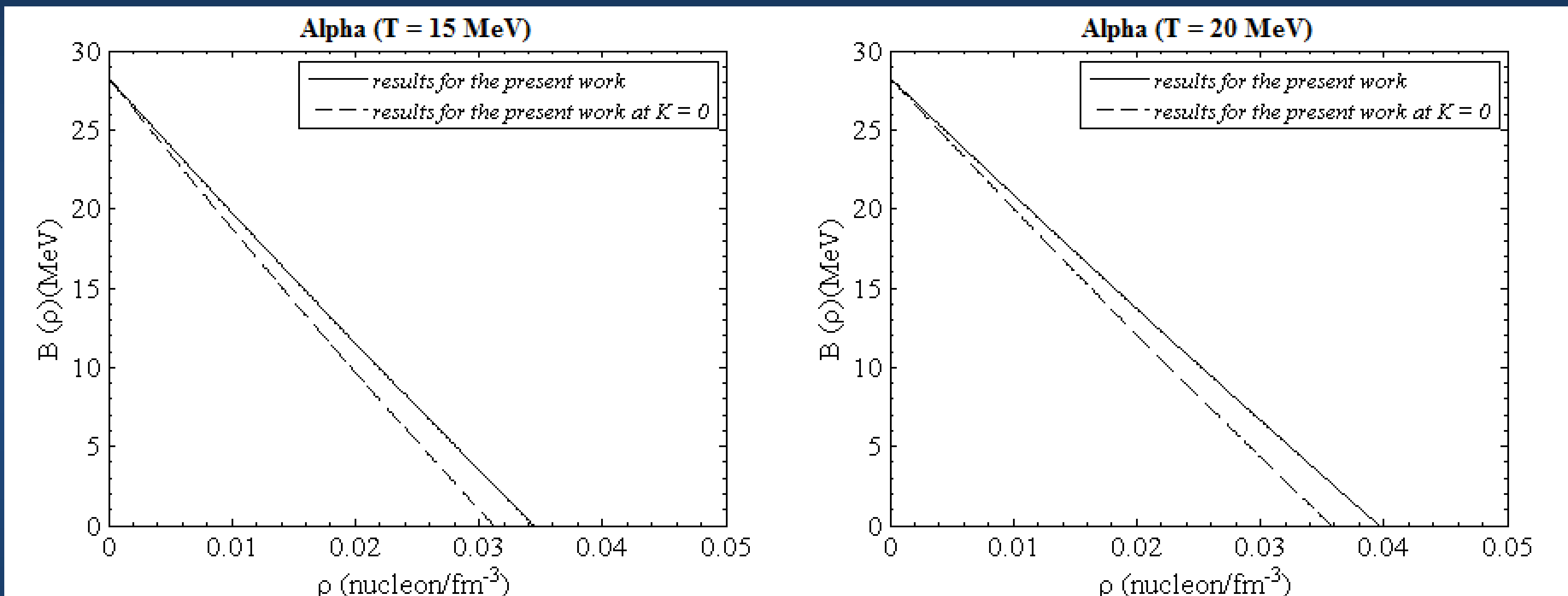
Results

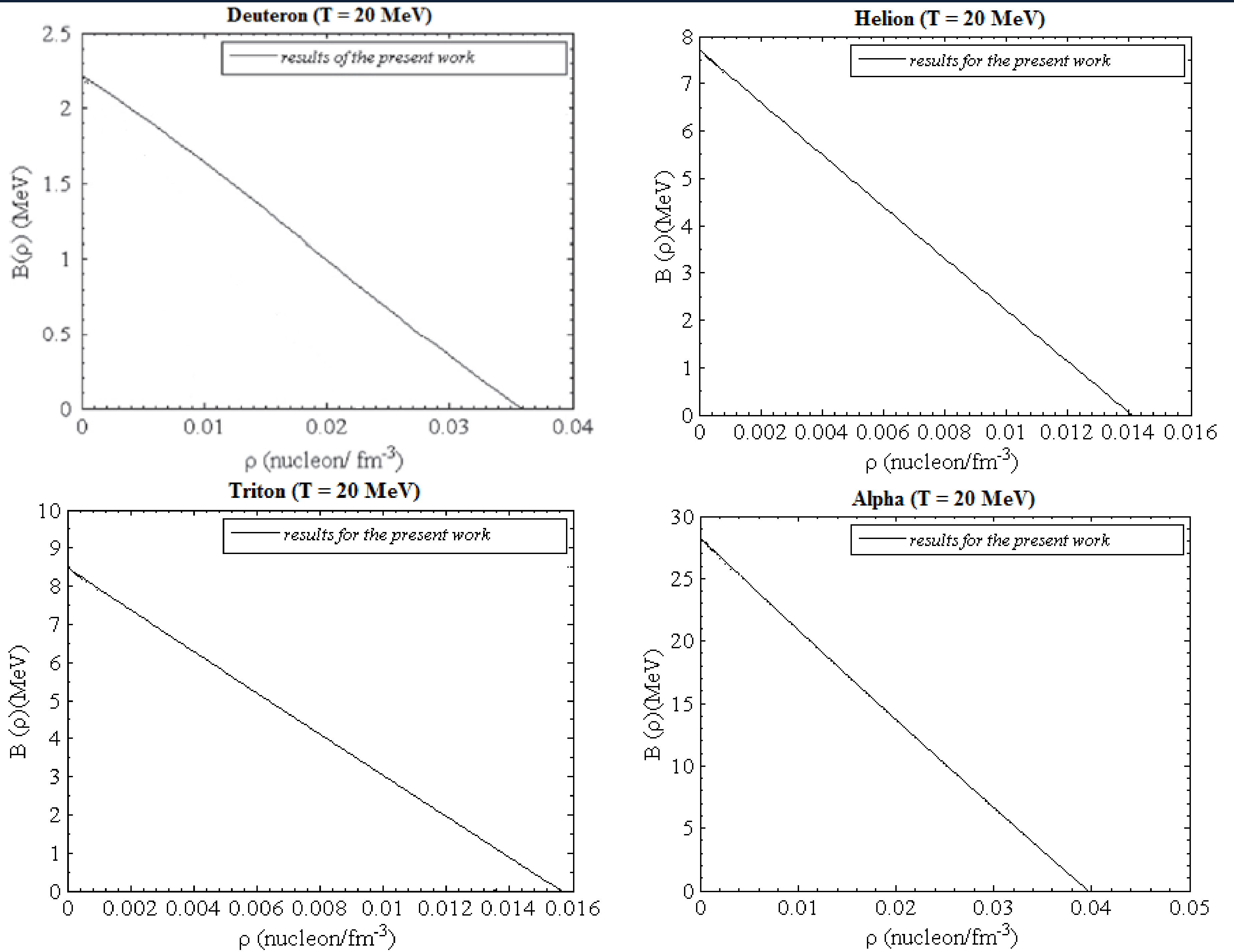
We used MATLAB to evaluate the expectation value of the binding energy as a function of the number density at different temperatures




Helion (T = 5 MeV)
Helion (T = 10 MeV)
Helion (T = 15 MeV)
Helion (T = 20 MeV)
results for the present work
results for the present work at K = 0
B (ρ)(MeV)
ρ (nucleon/fm-3)
0
1
2
3
4
5
6
7
8
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016


Triton (T = 5 MeV)
Triton (T = 10 MeV)
Triton (T = 15 MeV)
Triton (T = 20 MeV)
results for the present work
results for the present work at K = 0
B (ρ)(MeV)
ρ (nucleon/fm-3)
0 0.002 0.004 0.006 0.008 0.01 0.012 0.014 0.016
0 1 2 3 4 5 6 7 8 9 10


Alpha (T = 15 MeV)
results for the present work
results for the present work at K = 0
B (ρ)(MeV)
ρ (nucleon/fm-3)
Alpha (T = 20 MeV)
results for the present work
results for the present work at K = 0
B (ρ)(MeV)
ρ (nucleon/fm-3)



Deuteron (T = 20 MeV)
results of the present work
B(ρ) (MeV)
ρ (nucleon/ fm-3)
Helion (T = 20 MeV)
results for the present work
B (ρ)(MeV)
ρ (nucleon/fm-3)
Triton (T = 20 MeV)
results for the present work
B (ρ)(MeV)
ρ (nucleon/fm-3)
Alpha (T = 20 MeV)
results for the present work
B (ρ)(MeV)
ρ (nucleon/fm-3)

Review

- Regardless of the temperature, when clusters are moving within the surrounding medium ($\vec{K} \neq 0$) they can survive up to higher densities than that if they are at rest $(\vec{K} = 0)$. (Pauli blocking effect is less effective in this case).
- As the temperature increases the cluster can survive up to higher densities (Pauli blocking effect is less effective).
- As the mass of the cluster increases the cluster can survive up to higher densities.

Thanks for Your Attention

Questions???