Asset or Nothing Call

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Goals

- An asset-or-nothing call is an option where the buyer either gets the underlying (stock) at a certain date (maturity) or gets nothing, depending on whether the underlying price reaches a certain level or not. It is an example of a binary option.
- In this presentation we want discuss some basic features of the option and also calculate its value for the sake of profitability.

Overview

- Introduction
- Asset or Nothing Call
- Properties of Asset or Nothing Call
- The Value of Asset or Nothing Call
- Conclusion

Introduction

Binary Option

In finance, a binary option is a type of option in which the payoff can take only two possible outcomes, either some fixed monetary amount (or a precise predefined quantity or units of some asset) or nothing at all (in contrast to ordinary financial options that typically have a continuous spectrum of payoff).

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Asset or Nothing Call

- An asset-or-nothing binary option is not a usual call or put option. Asset-or-nothing binary option is basically a bet on the price of an asset at a certain date in the future. It is just a way to gamble

- If you purchase the option and win the bet, the option writer gives you the asset, so there are only two possible outcomes, this is why it is called a binary option.

Unlike other options where a call means the right to buy and a put the right to sell, an asset or nothing call is purchased when a trader believes the price of an asset will increase.

Properties of Asset or Nothing Call

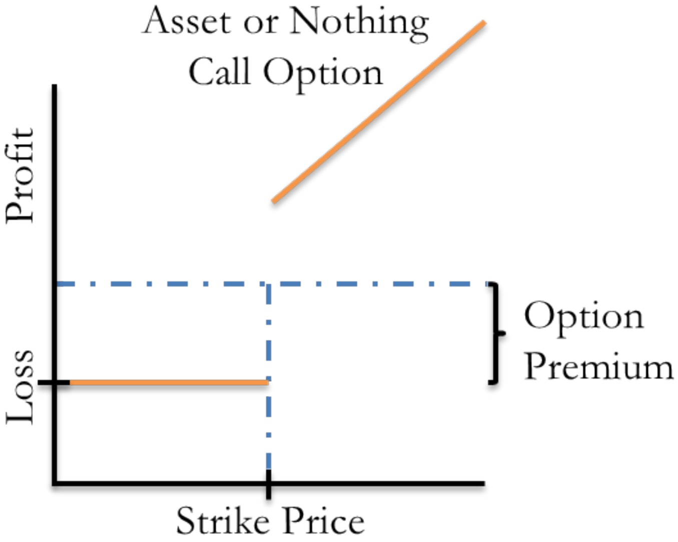
Payoff

For a call to make money, the price must be above the strike price at maturity. At expiration, the asset-or-nothing call option pays 0 if $S \leq K$ and pays S if $S > K$.
In other words, the asset-or-nothing call pays one unit of asset if the spot is above the strike at maturity

Method of Exercising

Asset-or-nothing options are usually European-style (i.e they are exercised on the expiration date).
It is traded at the stock exchange market like every other type of options.

Payoff for Asset or Nothing Call



The Value of Asset or Nothing Call

Notations

$$
\begin{aligned}
t &= \text{Current time}\\
K &= \text{Strike price}\\
T &= \text{Maturity time}\\
\sigma &= \text{Volatility}\\
r &= \text{Discounted rate}\\
S(t) &= \text{Current stock price}\\
S(T) &= \text{Stock price at maturity}\\
S^*(T) &= \text{Discounted stock price}\\
C(T) &= \text{Value of option at maturity}\\
C(t) &= \text{Current value of the option}
\end{aligned}
$$

The Value of Asset or Nothing Call

As we all know that the price of an asset is not constant, hence we include the volatility (the variation in the price of an asset).

The question now is what is the cost of the option?

The cost of the option is determined by factors including; stock price, time remaining until expiration (time value) and volatility.

The value of the option at maturity is given as;

$$C(T) = S(T).\mathbb{I}_{[S(T) \geq K]} = \begin{cases} S(T) & \text{if} \quad S(T) \geq K \\ 0 & \quad\quad \text{else} \end{cases}$$

The Value of Asset or Nothing Call

The value of money at hand today is different from the value of the same money in the future, we then introduce the discounted stock price. Hence the current value of an option $C(t)$ is the discounted expectation of the value at maturity:

$$C(t) = e^{-rT}\mathbb{E}_{\mathbb{P}}[C(T)].$$

By substituting $C(T)$, we have

$$C(t) = e^{-rT}\mathbb{E}_{\mathbb{P}}[S(T)\mathbb{I}_{[S(T)\geq K]}].$$

Discounted stock price can be evaluated in the present tense (i.e it is martingale) with the aid of the discounting factor e^{-rT}

$$S^*(T) = e^{-rT}S(T)$$

$$C(t) = \mathbb{E}_{\mathbb{P}}[S^*(T)\mathbb{I}_{[S^*(T)\geq e^{-rT}K]}]$$

The Value of Asset or Nothing Call

$S^*(T)$ follows a Geometry Brownian Motion and thus we can compute the expected value of $C(t)$ above by viewing $S^*(T)$ as a function of random variable $B(T)$ and use its density.

$$S^*(T) = S(t)e^{[-\frac{1}{2}\sigma^2 T + \sigma B(T)]} . \mathbb{I}_{[B(T) \geq a]}$$

The probability density of $B(T)$ is given by

$$\frac{1}{\sqrt{T}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x}{\sqrt{T}}\right)^2}$$

Using the probability density of $B(T)$ to evaluate the expectation,

$$C(t) = \mathbb{E}_{\mathbb{P}}[S^*(T)\mathbb{I}_{[S(T) \geq e^{-rT} K]}]$$

$$= \int_{x=a}^{\infty} S(t)e^{[-\frac{1}{2}\sigma T + \sigma x]} \frac{1}{\sqrt{T}\sqrt{2\pi}} \left(e^{-\frac{1}{2}\left(\frac{x}{\sqrt{T}}\right)^2}\right) dx$$

The Value of Asset or Nothing Call

In order to generate a standard normal distribution for easy evaluation, we simplify the integrand by changing of variable by making the change of variable

$$y = \frac{x}{\sqrt{T}}.$$

$C(t)$ therefore becomes

$$C(t) = S(t)\int_{y=\frac{x}{\sqrt{T}}}^{\infty} e^{[-\frac{1}{2}\sigma T+\sigma\sqrt{T}y]}\frac{1}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}y^2}\right)dy.$$

Rearranging the integrand, we get

$$C(t) = S(t)\int_{y=\frac{x}{\sqrt{T}}}^{\infty} \frac{1}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}(y-\sigma\sqrt{T})^2}\right)dy$$

The Value of Asset or Nothing Call

The integral is then transformed into a standard normal density by defining,

$$z = y - \sigma\sqrt{T}$$

$$C(t) = S(t)\int_{z=zlow}^{\infty} \frac{1}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}z^2}\right) dz$$

Where

$$zlow = \frac{a}{\sqrt{T}} - \frac{\sigma}{\sqrt{T}}$$

The integral is a standard normal distribution and hence we read from the normal distribution

$$C(t) = S(t)[1 - N(zlow)]$$

The normal distribution is symmetrical. Hence
$1 - N(zlow) = N(-zlow)$

$$C(t) = S(t)N(-zlow)$$

The Value of Asset or Nothing Call

Note that N denotes the standard normal The expression for $-zlow$ being a portion of the probability density function of a normal distribution, it is commonly denoted by

$$d_1 = \frac{\ln[\frac{S(t)}{K}] + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

Hence the price we pay for Asset or nothing call is given by;

$$C(t) = S(t)N(d_1)$$

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Explore


Thank you for listening !
I would love to answer questions
about my presentation !